

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \sin^3 x \cos x \, dx \quad u = \sin x$$

$$du = \cos x \, dx$$

$$\int_{\frac{\sqrt{2}}{2}}^{\frac{\sqrt{3}}{2}} u^3 \, du$$

$$\begin{aligned} \frac{1}{4} u^4 \Big|_{\frac{\sqrt{2}}{2}}^{\frac{\sqrt{3}}{2}} &= \frac{1}{4} \left(\frac{9}{16} - \frac{4}{16} \right) \\ &= \frac{1}{4} \left(\frac{5}{16} \right) \\ &= \boxed{\frac{5}{64}} \end{aligned}$$

$$\int \cos^3 x \, dx$$

$$\int \cos^2 x \cos x \, dx$$

$$\int (1 - \sin^2 x) \cos x \, dx \quad u = \sin x$$

$$\int 1 - u^2 \, du$$

$$u - \frac{1}{3} u^3 + C$$

$$\boxed{\sin x - \frac{1}{3} \sin^3 x + C}$$

$$\int \sin^5 2x \cos^4 2x dx$$

$$\underbrace{\int (\sin^4 2x \cos^4 2x) \sin 2x dx}_{\int \sin^2 2x \sin^2 2x \cos^4 2x \sin 2x dx}$$

$$\int (1 - \cos^2 2x)^2 \cos^4 2x \sin 2x dx$$

$$u = \cos 2x$$

$$du = -2 \sin 2x dx$$

$$-\frac{1}{2} \int (1-u^2) u^4 du$$

$$-\frac{1}{2} \int (1-2u^2+u^4) u^4 du$$

$$-\frac{1}{2} \int u^4 - 2u^6 + u^8 du$$

$$-\frac{1}{2} \left(\frac{1}{5}u^5 - \frac{2}{7}u^7 + \frac{1}{9}u^9 \right) + C$$

$$\boxed{-\frac{1}{2} \left(\frac{1}{5} \cos^5 2x - \frac{2}{7} \cos^7 2x + \frac{1}{9} \cos^9 2x \right) + C}$$

$$\int \cos^2 x \, dx$$

$$\frac{1}{2} \int (1 + \cos 2x) \, dx$$

$$\frac{1}{2} \left(x + \frac{1}{2} \sin 2x \right) + C$$

$$\frac{1}{2}x + \frac{1}{4}\sin 2x + C$$

Reduction Identities

$$\sin^2 x = \frac{1}{2} (1 - \cos 2x)$$

$$\cos^2 x = \frac{1}{2} (1 + \cos 2x)$$

MEMORIZE

SUMMARY : POWERS OF SINE/COSINE

IF ANY EXPONENT IS ODD :

① ISOLATE A SINGLE POWER OF SINE OR COSINE

② REWRITE EVERYTHING ELSE IN TERMS OF THE OTHER TRIG FXN

③ USE U-SUB TO INTEGRATE

PYTHAGOREAN
IDENTITIES

IF ALL EXPONENTS ARE EVEN:

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

① USE REDUCTION IDENTITIES : $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$

② YOU MAY HAVE TO USE THEM MORE THAN ONCE...

(YOU MAY ALSO USE PYTHAG. IDENTITIES...)

$$\int \sin^4 x \, dx$$

$$\int \sin^2 x \sin^2 x \, dx$$

$$\int \frac{1}{2}(1-\cos 2x) \cdot \frac{1}{2}(1-\cos 2x) \, dx$$

$$\frac{1}{4} \int 1 - 2\cos 2x + \cos^2 2x \, dx$$

$$\frac{1}{4} \int 1 - 2\cos 2x + \frac{1}{2}(1 + \cos 4x) \, dx$$

$$\rightarrow \frac{1}{4} \int \frac{3}{2} - 2\cos 2x + \frac{1}{2}\cos 4x \, dx$$

$$\frac{1}{4} \left(\frac{3}{2}x - \sin 2x + \frac{1}{8}\sin 4x \right) + C$$

$$\frac{3}{8}x - \frac{1}{4}\sin 2x + \frac{1}{32}\sin 4x + C$$

$$\begin{aligned}
 & \int \sin^4 x \cos^4 x dx \\
 & \int \left[\frac{1}{2} (1 - \cos 2x) \right]^2 \left[\frac{1}{2} (1 + \cos 2x) \right]^2 dx \\
 & \frac{1}{16} \int [(1 - \cos 2x)(1 + \cos 2x)]^2 dx \\
 & \frac{1}{16} \int (1 - \cos^2 2x)^2 dx \\
 & \frac{1}{16} \int (\sin^2 2x)^2 dx \\
 & \frac{1}{16} \int \left(\frac{1}{2} (1 - \cos 4x) \right)^2 dx
 \end{aligned}$$

$$\begin{aligned}
 & \frac{1}{64} \int 1 - 2\cos 4x + \cos^2 4x dx \\
 & \frac{1}{64} \int 1 - 2\cos 4x + \frac{1}{2} (1 + \cos 8x) dx \\
 & \frac{1}{64} \int \frac{3}{2} - 2\cos 4x + \frac{1}{2} \cos 8x dx \\
 & \boxed{\frac{1}{64} \left(\frac{3}{2}x - \frac{1}{2} \sin 4x + \frac{1}{16} \sin 8x \right) + C}
 \end{aligned}$$

P 450 #1-19 ODD
ANTON

